Superconductivity - the Higgs mechanism and the fate of the Goldstone mode.

Superconductor - charged superfluid

\[ S = S_\theta + S_{EM} \]

\[ S_\theta = \int dt \, d^3x \, \frac{n_s}{2\hbar^*} \left[ \frac{1}{C_s^2} (D_\mu \theta)^2 - (D_\mu \theta)^2 \right] \]

where \( D_\mu \theta = \partial_\mu \theta - e^* A_\mu \) with \( e^* = 2e \)

\[ S_{EM} = \frac{1}{2\mu} \int dt \, d^3x \left( \frac{E}{c} \right)^2 - B^2 \]

\[ E_i = \partial_t A_i - \partial_i A_\tau \]

\[ B_i = \epsilon_{ijk} \partial_j A_k \]

\( S \) is invariant under local \( U(1) \) gauge transformations:

\[ \theta \rightarrow \theta + \omega, \quad A_\mu \rightarrow A_\mu + e^* \partial_\mu \omega \]

The Goldstone phase \( \theta \) can now be absorbed into the gauge field \( A_\mu \) via a gauge transformation.

\[ S = \int dt \, d^3x \left[ \frac{1}{2} \frac{n_s}{\hbar^*} e^* \left\{ \left( \frac{A_t}{C_s} \right)^2 - A_i^2 \right\} + \frac{1}{2\mu} \left\{ \left( \frac{E}{c} \right)^2 - B^2 \right\} \right] \]

New length scale intrinsic to superconductors:

London penetration length \( \lambda_L \).
\[
\frac{1}{\hbar^2} = \frac{\hbar s}{\hbar^2} e^{-z} \Rightarrow h_c = \sqrt{\frac{\hbar^*}{\hbar s (e^*)^2}}
\]

physical meaning will be clarified later.

We found a theory of a massive U(n) gauge field \( \to \) photon has a finite mass (short-ranged) \( \Rightarrow \) screening.

This is called the \textit{Lego} mechanism.

massless Q.Goldstne and photon disappeared.

Introduce \( g \) and \( j^i \) from \( S_0 \):

\[
\mathcal{S} = -\frac{\delta S_0}{\delta \mathbf{A}} = -\frac{1}{\sqrt{\varepsilon_s \chi^2}} \mathbf{A}_t
\]

\[
\mathcal{J} = -\frac{\delta S_0}{\delta \mathbf{A}_i} = +\frac{1}{\sqrt{\varepsilon_s \chi^2}} \mathbf{A}_i \Rightarrow \text{Lump equation}
\]

Now use it in EoM \( \frac{\delta S}{\delta \mathbf{A}_p} = 0 \):

\[
\frac{\delta S}{\delta \mathbf{A}_t} = 0 \Rightarrow \text{Gauss law} \hspace{1cm} \mathbf{E} \cdot \mathbf{A} = \mathbf{P}
\]

\[
\frac{\delta S}{\delta \mathbf{A}_i} = 0 \Rightarrow \text{Ampere law}
\]

\[
\frac{1}{\mu} \left( \frac{1}{c^2} \partial_t \mathbf{E} - \nabla \times \mathbf{B} \right) = \mathbf{j}^i
\]
Write the Ampere law in terms of $\vec{A}$, use the continuity equation $\nabla \cdot j + \frac{d\rho}{dt} = 0$ to find

$$\left( \square \vec{A} - \frac{1}{4\pi} \frac{1}{\ell^2} \vec{A} \right) = \left[ 1 - \left( \frac{c_s}{c} \right)^2 \right] \nabla \left( \nabla \cdot \vec{A} \right)$$

Solve in Fourier space:

a) transverse modes $\nabla \cdot \vec{A} = 0$

$$\omega^2 = \left( \omega_0 \frac{c_s}{c} \right)^2 + (q \frac{c_s}{c})^2$$

$$\omega \propto \text{speed of light} \quad \phi$$

static case ($\partial_t \phi = 0$) $\Rightarrow$ Meissner effect

$$\nabla \times \vec{A}_T = \frac{1}{2\pi \ell^2} \vec{A}_T = 0 \Rightarrow \vec{A}_T \sim e^{-x/\ell}$$

magnetic field is expelled from the superconductor

b) longitudinal modes $\nabla \times \vec{A} = 0 \Rightarrow \text{Rus almost cancels } \delta^2 \phi \text{ on the RHS}$

$$\omega^2 = \left( \omega_0 \frac{c_s}{c} \right)^2 + (\rho \frac{c_s}{c})^2$$

speed of sound

this is the plasmon mode which goes into metallic plasmon in the normal state $T > T_c$
In non-relativistic SC, \( \mathbf{E} \)- and \( \mathbf{B} \)-modes have very different group velocity of propagation. In a Lorentz-invariant SC (\( C_{\mu} = C \)), the RES of the Ampere law vanishes \( \Rightarrow \) both \( \mathbf{E} \)- and \( \mathbf{B} \)-modes propagate with the velocity of light.

Superconductors are not covered in detail in this lecture, more in books Tinkham, de Gennes.

1) Type I vs Type II SC
\[
(\lambda_c < \frac{1}{5} \xi) \quad (\lambda_c > \frac{1}{12} \xi)
\]
depends on the surface energy of a domain wall SC-N, vortex crystals

2) Vortices are exponentially localized
\[
\psi_{\text{sc}} \sim \exp \left( -\frac{1}{\lambda_c} \right)
\]
and a magnetic flux
\[
\Phi \nu = \frac{\hbar}{e} = \frac{\Phi}{e}
\]
leads to every vortex.